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Field-theoretical approach to particle oscillations in absorbing matter

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Abstract. The ab oscillations in absorbing matter are considered. The standard model based on an optical potential does not describe the total ab transition probability as well as the channel corresponding to absorption of the b-particle. We calculate directly the off-diagonal matrix element in the framework of the field-theoretical approach. Contrary to the one-particle model, the final-state absorption does not tend to suppress the channels mentioned above; or, similarly, calculation with a Hermitian Hamiltonian leads to an increase of the corresponding values. Our approach reproduces all the results on the particle oscillations; however, it is oriented to the description of the above-mentioned channels. Also we touch on the general problems of the theory of reactions and decays; in particular, on infrared divergences. The approach in our study is infrared-free. For the correct construction and verification of the models of complicated processes, we propose the use of the low-density limit for the intermediate-state particle.

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1 Introduction

The theory of ab oscillations is based on the one-particle model [1–5]. The interaction of particles a and b with the matter is described by the potentials $U_{a,b}$. $\mathrm{Im}\, U_b$ is responsible for the loss of b-particle intensity. The wave functions $\Psi_{a,b}$ are given by the equations of motion. The index of refraction, the forward scattering amplitude $f(0)$ and the potential are related to each other, so later on the standard approach is referred to as the potential model.

The description of absorption by means of $\mathrm{Im}\, U_b$ is at least imperfect and should be partially revised. As an illustration, let us consider the case of strong b-particle absorption. Instead of a periodic process, we get two-step system decay:

$$(a\text{–medium}) \to (b\text{–medium}) \to f. \qquad (1)$$

Here (b–medium) $\to f$ represents the b-particle absorption. The potential model does not describe this process as well as the total ab transition probability (see [6–8] and the next section). It describes the probability of finding a b-particle only.

By way of a specific example we consider the $n\bar{n}$ transitions in a medium followed by annihilation [9–16]:

$$(n\text{–medium}) \to (\bar{n}\text{–medium}) \to M, \qquad (2)$$

where M are the annihilation mesons. The qualitative picture of the process is as follows. The free-space $n\bar{n}$ transition comes from the exchange of Higgs bosons with a mass $m_H > 10^5$ GeV [10, 11]. From the dynamical point of view this is a momentary process: $\tau_c \sim 1/m_H < 10^{-29}$ s. The antineutron annihilates in a time $\tau_a \sim 1/\Gamma$, where Γ is the annihilation width of $\bar{n}$ in the medium. We deal with a two-step process with the characteristic time $\tau_2 \sim \tau_a$.

The potential model describes the probability of finding an antineutron only, whereas a main contribution is given by the process (2), because $\bar{n}$ annihilates in a time τ_a. In the following we consider the $n\bar{n}$ transitions, since the final-state absorption in this case is extremely strong.

So the model should describe two-step processes like (1) and (2). On the other hand, as the absorption Hamiltonian tends to zero, the well-known results on particle oscillations should be reproduced. This program is realized below. Also we present here an elaborated derivation of the lower limit on the $n\bar{n}$ oscillation time and discuss in some detail the uncertainties connected with medium corrections.

The $n\bar{n}$ transitions in the medium are the ideal instrument for the study of the intermediate- and final-state interactions [8]. So, besides the oscillations proper, we touch on the general problems of the theory of reactions and decays, namely the following.

1) The infrared-free approach and its connection with the S-matrix theory.

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2) The role of final-state absorption and decay.
3) The particle self-energy and competition between elastic and inelastic interactions in the intermediate state.
4) The time-dependence of the process.

Some of these points are more important than the lower limit on the $n\bar{n}$ oscillation time; for example, the infrared-free approach.

The paper is organized as follows. In the next section, we recall the main results and point out the chief drawback of the potential model. In this model the $n\bar{n}$ transition probability depends critically on the antineutron self-energy Σ. In the field-theoretical approach a similar thing takes place (Sects. 3 and 4). Because of this, particular attention is given to the suppression mechanism and to the origin of Σ. In Sect. 4 we arrive at the conclusion that $\Sigma = 0$. An important effect of the competition between scattering and annihilation of $\bar{n}$ in the intermediate state is studied as well. The main calculations are performed in Sects. 5 and 6. If $\Sigma = 0$, the S-matrix amplitudes contain the infrared singularity. For solving the problem, the approach with a finite time interval (FTA) [17] is used. It is infrared-free. First of all, we verify the FTA by the example of an exactly solvable potential model. The FTA reproduces all the results on the particle oscillations ($\nu_a\nu_b$, $n\bar{n}$, etc.). In Sect. 6 the process (2) and the process shown in Fig. 8b are calculated. The linkage between the S-matrix theory and FTA is studied as well. In Sect. 7 we complete the calculation of process (2). Also we arrive at the conclusion that for the processes with zero momentum transfer the problem should be formulated on a finite time interval. The results are summarized and discussed in Sect. 8. The limiting cases and the effects of absorption and coherent forward scattering are discussed as well. Section 9 contains the conclusion.

2 Potential model

In this section we touch briefly on the main results and the range of applicability of the potential model in the case of $n\bar{n}$ transitions. The chief drawback of this model is given as well.

Let $U_n = \text{const.}$ and $U_{\bar{n}} = \text{const.}$ be the neutron potential and the optical potential of $\bar{n}$, respectively. The background field U_n is included in the unperturbed Hamiltonian H_0. The interaction Hamiltonian has the form

$$\begin{aligned} \mathcal{H}_{\rm I} &= \mathcal{H}_{n\bar{n}} + \mathcal{H}\,, \\ \mathcal{H}_{n\bar{n}} &= \epsilon\bar{\Psi}_{\bar{n}}\Psi_n + \text{H.c.}\,, \\ \mathcal{H} &= (\text{Re}\,V + \text{i}\,\text{Im}\,V)\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}}\,, \\ V &= U_{\bar{n}} - U_n = \text{Re}\,U_{\bar{n}} - U_n - \text{i}\Gamma/2\,. \end{aligned} \tag{3}$$

Here $\mathcal{H}_{n\bar{n}}$ and $\mathcal{H}$ are the Hamiltonians of the $n\bar{n}$ transition [13,14] and the $\bar{n}$–medium interaction, respectively; ϵ is a small parameter with $\epsilon = 1/\tau_{n\bar{n}}$, where $\tau_{n\bar{n}}$ is the free-space $n\bar{n}$ oscillation time.

The model can be realized by means of the diagram technique [6,7,15] or the equations of motion [12–14,16]:

$$\begin{aligned} (\text{i}\partial_t - H_0)n(x) &= \epsilon\bar{n}(x)\,, \\ (\text{i}\partial_t - H_0 - V)\bar{n}(x) &= \epsilon n(x)\,, \\ H_0 &= -\nabla^2/2m + U_n\,, \end{aligned} \tag{4}$$

and $\bar{n}(0,\mathbf{x}) = 0$. For $V = \text{const.}$ in the lowest order in ϵ, the probability of finding an $\bar{n}$ in a time t is found to be

$$W_{\bar{n}}(t) = \frac{\epsilon^2}{|V|^2}\left[1 - 2\cos(\text{Re}\,Vt)\text{e}^{-\Gamma t/2} + \text{e}^{-\Gamma t}\right]. \tag{5}$$

In the following we focus on the most important case: $\Gamma t \gg 1$. Then $W_{\bar{n}}(t) \sim \epsilon^2/|V|^2 \ll 1$. The total $n\bar{n}$ transition probability $W_{\rm t}^{\rm pot}$ (more precisely, the probability of finding an $\bar{n}$ or annihilation products) is given by

$$\begin{aligned} W_t^{\rm pot}(t) &= 1 - |\langle n(0)|n(t)\rangle|^2 \\ &\approx 2\epsilon^2 t\frac{\Gamma/2}{(\text{Re}\,V)^2 + (\Gamma/2)^2} \approx \frac{4\epsilon^2 t}{\Gamma}\,. \end{aligned} \tag{6}$$

The index "pot" signifies that the non-Hermitian Hamiltonian (3) is used.

The free-space $n\bar{n}$ transition probability W_f is $W_f = \epsilon^2 t^2$. Comparing with (6), one obtains the suppression factor $R_{\rm pot}$:

$$R_{\rm pot} = \frac{W_{\rm t}^{\rm pot}}{W_f} = \frac{\Gamma}{|V|^2 t} \sim \frac{1}{|V|t} \ll 1\,. \tag{7}$$

The energy gap $\delta E = V$ leads to very strong suppression of the $n\bar{n}$ transition in the medium and changes the functional structure of the result: $W_f \sim t^2 \to W_t^{\rm pot} \sim t$. Because of this, particular attention is given to the suppression mechanism and to the origin of δE.

The energy gap is the antineutron self-energy: $\delta E = V = \Sigma$. Indeed, the result (6) can be obtained by means of the diagram technique. In the potential model $W_{\rm t}^{\rm pot} = 1 - \exp(-\Gamma_{\rm t}^{\rm pot}t) \approx \Gamma_{\rm t}^{\rm pot}t$. The total process width $\Gamma_{\rm t}^{\rm pot}$ is easily calculated [6,7]:

$$\Gamma_{\rm t}^{\rm pot} = \frac{1}{T_0}(1 - |S_{ii}|^2) \approx -2\,\text{Im}\,\epsilon G^{\rm pot}\epsilon\,, \tag{8}$$

$$\begin{aligned} G^{\rm pot} &= \frac{1}{\epsilon_n - \mathbf{p}_n^2/2m - U_{\bar{n}}} \\ &= \frac{1}{(\epsilon_n - \mathbf{p}_n^2/2m - U_n) - V} = -\frac{1}{V}\,. \end{aligned} \tag{9}$$

Here $G^{\rm pot}$ is the antineutron propagator, $p = (\epsilon_n, \mathbf{p}_n)$ is the neutron 4-momentum; $\epsilon_n = \mathbf{p}_n^2/2m + U_n$, T_0 is the normalization time, $T_0 \to \infty$. We thus see that the suppression factor $R_{\rm pot}$ is due to the antineutron self-energy $\Sigma = V$.

Consider now the range of applicability of the model (4). The total $n\bar{n}$ transition probability $W_{\rm t}$ is

$$W_{\rm t} = W_{\bar{n}} + W_{\rm a}\,, \tag{10}$$

where $W_{\rm a}$ is the probability of finding the annihilation mesons (i.e. the probability of the process (2)). The potential model correctly describes the $W_{\bar{n}}$. However, for the calculation of $W_{\rm t}$ and $W_{\rm a}$ it is inapplicable. In the one-particle model the total process width $\Gamma_{\rm t}^{\rm pot}$ is obtained by means of (8), which follows from the unitarity condition. The same is true for (6). Since the Hamiltonian (3) is non-Hermitian (in the first approximation one can put $\mathrm{Re}\, V = 0$), the S-matrix is non-unitary and (8) and (6) are invalid [6–8]. The condition of probability conservation,

$$1 = |S_{ii}|^2 + \sum_{f \neq i} |T_{fi}|^2 \,,$$

can be used only if the S-matrix is unitary or unitarized.

It must be emphasized that in the problem under study the unitarity of S-matrix is of particular importance because $\mathrm{Im}\, V$ enters the leading diagram and plays a crucial role (see (9)).

As a result, in the potential model the effect of final-state absorption acts in the opposite (wrong) direction [8], which is not surprising, since the unitarity condition and non-unitary S-matrix are mutually incompatible. The condition $SS^+ = 1$ is applied to the essentially non-unitary S-matrix. (The potential model does not describe the $W_{\rm t}$ at all. The non-unitarity is only a formal manifestation of this fact.) As shown below, the potential model also does not describe the competition between scattering and annihilation of $\bar{n}$ in the intermediate state and the time dependence of the process (2). The greater $|\mathrm{Im}\, V|$, the greater the error in the $W_{\rm t}^{\rm pot}$ and $W_{\rm a}$. So (6) and (8) are incorrect. A direct calculation of the antineutron absorption (process (2)) is called for.

3 Free-space process

First of all we consider the free-space $\bar{n}N$ annihilation (see Fig. 1a) and the free-space process

$$n + N \to \bar{n} + N \to M \,, \tag{11}$$

shown in Fig. 1b.

The matrix element of the S-matrix $T_{\rm a}$ and the amplitude $M_{\rm a}$ corresponding to Fig. 1a are defined as

$$\begin{aligned} \mathrm{i}T_{\rm a} &= \langle M | T \exp\left(-\mathrm{i} \int \mathrm{d}x \mathcal{H}_{\bar{n}N}(x) \right) - 1 | \bar{n}N \rangle \\ &= N_{\rm a} (2\pi)^4 \delta^4 (p_f - p_i) M_{\rm a} \,. \end{aligned} \tag{12}$$

Here $\langle M|$ represents the annihilation mesons, $\mathcal{H}_{\bar{n}N}$ is the Hamiltonian of the $\bar{n}N$ interaction, $N_{\rm a}$ includes the normalization factors of the wave functions.

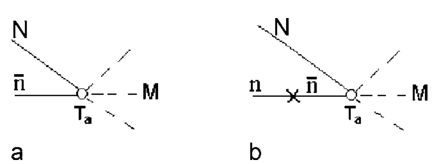

Fig. 1. a Free-space $\bar{n}N$ annihilation. **b** Free-space process $n + N \to \bar{n} + N \to M$

$T_{\rm a}$ and $M_{\rm a}$ involve all the $\bar{n}N$ interactions followed by annihilation including $\bar{n}N$ *rescattering in the initial state*. The same is true for the subprocess of $\bar{n}N$ annihilation involved in Fig. 1b: the block $T_{\rm a}$ should contain all the $\bar{n}N$ interactions followed by annihilation.

We write the formulas corresponding to Fig. 1b. The interaction Hamiltonian is

$$\mathcal{H}_{\rm I} = \mathcal{H}_{n\bar{n}} + \mathcal{H}_{\bar{n}N} \,. \tag{13}$$

Formally, in lowest order in $\mathcal{H}_{n\bar{n}}$ the amplitude of the process (11) is given by

$$\begin{aligned} M_{1\rm b} &= \epsilon G_0 M_{\rm a} \,, \\ G_0 &= \frac{1}{\epsilon_n - \mathbf{p}_n^2/2m} \,. \end{aligned} \tag{14}$$

Here G_0 is the antineutron propagator. Since $M_{\rm a}$ contains all the $\bar{n}N$ interactions followed by annihilation, G_0 is bare. We emphasize this fact as it gives insight into the origin of Σ.

Due to the zero momentum transfer in the $n\bar{n}$ transition vertex, the 4-momenta of n and $\bar{n}$ are equal. The pre- and post-$n\bar{n}$ conversion spatial wave functions of the system coincide: $|Nn_p\rangle_{\rm sp} = |N\bar{n}_p\rangle_{\rm sp}$. Actually this is true for any neutron state (for any nuclear model).

For the time being we do not go into the singularity $G_0 \sim 1/0$. It results from the zero momentum transfer in the vertex corresponding to $\mathcal{H}_{n\bar{n}}$. The value of Σ is disconnected with $\mathcal{H}_{n\bar{n}}$ and we want to separate these problems. The general consideration is given in Sect. 6.

4 $n\bar{n}$ transitions in the medium

In this section the origin of Σ (energy gap) is studied in the framework of microscopic theory. It is shown that the value of Σ is uniquely determined by the definition of the annihilation amplitude of $\bar{n}$ in the medium. It turns out that for a realistic definition $\Sigma = 0$. Also we consider the competition between scattering and annihilation of $\bar{n}$ in the intermediate state.

4.1 S-matrix approach

Let us consider the process (2). (The $n\bar{n}$ transitions with $\bar{n}$ in the final state are considered in the next section.) We use the scheme identical to that for process (11) with the substitution $\bar{n}N \to (\bar{n}$–medium). The background field U_n is included in the neutron wave function (Hamiltonian H_0); the quadratic terms $\mathcal{H}_{n\bar{n}}$ are included in $\mathcal{H}_{\rm I}$:

$$\begin{aligned} \mathcal{H}_{\rm I} &= \mathcal{H}_{n\bar{n}} + \mathcal{H} \,, \\ H(t) &= \int \mathrm{d}^3 x \mathcal{H}(x) \,. \end{aligned} \tag{15}$$

Here H is the Hermitian Hamiltonian of the $\bar{n}$–medium interaction. The sole physical distinction with the model (4) is in the Hamiltonian $\mathcal{H}$. Recall that the potential model does not describe the processes (2) and (11) [6–8].

The amplitude of the process, M_2, is uniquely determined by the Hamiltonian (15):

$$M_2 = \epsilon G_0^m M_{\rm a}^m\,,\qquad G_0^m = \frac{1}{\epsilon_n - \mathbf{p}_n^2/2m - U_n} \tag{16}$$

(see Fig. 2a). The matrix element of the S-matrix $T_{fi}^{\bar{n}}$ and the amplitude of antineutron annihilation in the medium $M_{\rm a}^m$ are

$$\mathrm{i}T_{fi}^{\bar{n}} = \langle f|T^{\bar{n}}|0\bar{n}_p\rangle = N(2\pi)^4\delta^4(p_f - p_i)M_{\rm a}^m\,,\qquad T^{\bar{n}} = T\exp\left(-\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}tH(t)\right) - 1 \tag{17}$$

(compare with (12)). Here $|0\bar{n}_p\rangle$ is the state of the medium containing the $\bar{n}$ with the 4-momentum p, $\langle f|$ denotes the annihilation products, N includes the normalization factors.

The definition of the annihilation amplitude $M_{\rm a}^m$ through (17) is natural. If the number of particles of the medium is equal to unity, (17) goes into (12). The annihilation width Γ is expressed through $M_{\rm a}^m$: $\Gamma \sim \int \mathrm{d}\Phi|M_{\rm a}^m|^2$. Since H appears only in the $M_{\rm a}^m$, the antineutron propagator G_0^m is bare. In the next section we perform a rigorous calculation of M_2.

It is important that $M_2 \sim M_{\rm a}^m$. The value of Γ and corrections to $M_{\rm a}^m$ (if they are possible) have little effect on the results.

Construct now the model with the dressed propagator (see Fig. 2b). In the Hamiltonian $\mathcal{H}$ we separate out the real potential $V = \mathrm{Re}\,U_{\bar{n}} - U_n$,

$$\mathcal{H} = V\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} + \mathcal{H}'\,, \tag{18}$$

and include it in the antineutron Green function

$$G^m = G_0^m + G_0^m V G_0^m + \cdots = \frac{1}{(1/G_0^m) - V} = -\frac{1}{V}\,. \tag{19}$$

Then

$$M_2 = \epsilon G^m M'_{\rm a}\,, \tag{20}$$
$$G^m M'_{\rm a} = G_0^m M_{\rm a}^m\,. \tag{21}$$

The propagator G^m is dressed: $\Sigma = V \neq 0$. According to (21), the expressions for the propagator and vertex function are uniquely connected (if $H_{\rm I}$ is fixed). The "amplitude" $M'_{\rm a}(V, H')$ should describe the annihilation. However, below is shown that $M'_{\rm a}$ and model (20) are unphysical.

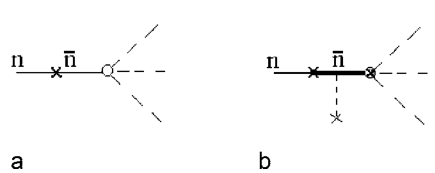

Fig. 2. a $n\bar{n}$ transition in the medium followed by annihilation. The antineutron annihilation is shown by a *circle*. **b** Same as **a** but the antineutron propagator is dressed (see text)

We recall that the amplitude $M_{\rm a}$ involves all the $\bar{n}N$ interactions followed by annihilation including rescattering in the initial state. Similarly, $M_{\rm a}^m$ involves all the $\bar{n}$–medium interactions followed by annihilation including the antineutron rescattering in the initial state. Compare now the left- and right-hand sides of (21).

From the physical point of view, model (20) has no justification because of the following reasons.

1) If the number of particles of medium n is equal to unity, the model (20) does not describe the free-space process (11) because (14) contains the bare propagator.

2) The observable values (Γ for example) are expressed through $M_{\rm a}^m$ and not $M'_{\rm a}$. Compared to $M_{\rm a}^m$, $M'_{\rm a}$ is truncated because the portion of the Hamiltonian H is included in G^m. $M'_{\rm a}$ does not have a physical meaning.

(The formal expression for the dressed propagator should contain the annihilation loops as well. In this case the statements given in 1) and 2) are only enhanced.)

3) Equations (19) and (20) mean that the annihilation is turned on upon forming of the self-energy part $\Sigma = V$ (after multiple rescattering of $\bar{n}$). This is counter-intuitive, since at low energies [18–20]

$$r = \frac{\sigma_{\rm ann}^{\bar{n}p}}{\sigma_{\rm el}^{\bar{n}p}} > 2.5 \tag{22}$$

and the inverse picture is in order: in the first stage of the $\bar{n}$–medium interaction the annihilation occurs.

Realistic *competition* between scattering and annihilation should be taken into account. Both scattering and annihilation vertices should occur on equal terms in $M_{\rm a}^m$ or G^m. According to 1) and 2) the latest possibility should be excluded. In line with the physical meaning of $M_{\rm a}^m$ and M_2, the amplitude (16) allows for the above-mentioned effect (see Sect. 4.3).

The structure with a dressed propagator like (20) arises naturally if V and $\mathcal{H}'$ are the principally different terms and the vertex function does not depend on V. In the problem under consideration this is not the case. This is evident from the formal expansion of the T-operator

$$T\exp\left(-\mathrm{i}\int \mathrm{d}x\big(V\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} + \mathcal{H}'\big)\right). \tag{23}$$

It is significant that even the non-realistic model (20) gives reinforcement in comparison with the potential model [6, 7]:

$$\frac{W_{\rm a}}{W_{\rm t}^{\rm pot}} = 1 + \left(\frac{\Gamma/2}{V}\right)^2 > 1\,, \tag{24}$$

because for the model (20) the probability of process (2) is

$$W_{\rm a} \sim \Gamma \tag{25}$$

instead of $W_{\rm t}^{\rm pot} \sim 1/\Gamma$.

To summarize, the introduction of the dressed propagator G^m (energy gap) into the model for the process entails an uncertainty of the vertex function $M'_{\rm a}$. The all-important effect of the competition is not taken into account. The limiting case $n=1$ is not reproduced. $M'_{\rm a}$ is unknown and unphysical.

We do not see the reasons for the existence of the field V, which should be included into G^m, and thus the antineutron propagator is bare. For the process shown in Fig. 8b the propagator is bare as well. Essentially, this fact governs the situation. Below we assume the definition (17) and consequently the model with the bare propagator (16).

4.2 Simplest model

The fact that the antineutron propagator is bare is obvious in the model containing the annihilation vertex only. We consider Fig. 1a. Assume that

$$\mathcal{H}_{\bar{n}N} = \Phi^*_M g_{\rm a} \Psi_{\bar{n}} \Psi_N \,, \tag{26}$$

where Φ_M denotes the fields of mesons. The diagrams of the $\bar{n}N$ annihilation are shown in Fig. 3.

Similarly, for the $\bar{n}$–medium annihilation we take

$$\mathcal{H} = \mathcal{H}_{\rm a} = \sum_i \Phi^*_M g_{\rm a} \Psi_{\bar{n}} \Psi_{N_i} \,. \tag{27}$$

The corresponding diagrams are shown in Fig. 4.

Consider now the process (2) using the same Hamiltonian $\mathcal{H}_{\rm a}$. The diagram is shown in Fig. 5; the Hamiltonian is given by (15), where $\mathcal{H}=\mathcal{H}_{\rm a}$. The antineutron propagator is bare. The questions connected with the self-energy part do not arise in principle, because $\mathcal{H}_{\rm a}$ must appear only in the $T^{\bar{n}}_{fi}$ (see Fig. 4). The block $T^{\bar{n}}_{fi}$ is described by (17) and (27).

In view of (22), the models like (27) are reasonable, and so it seems obvious that the antineutron propagator is bare.

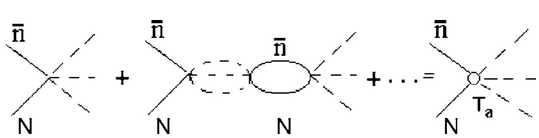

Fig. 3. $\bar{n}N$ annihilation. The interaction Hamiltonian is given by (26)

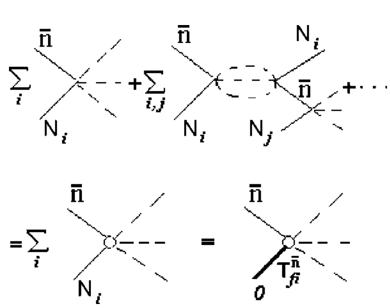

Fig. 4. Antineutron annihilation in the medium

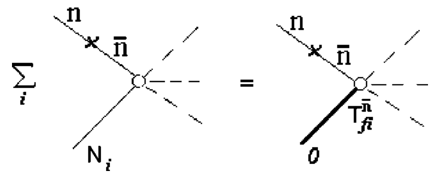

Fig. 5. $n\bar{n}$ transition in the medium followed by annihilation

4.3 Scattering and annihilation of $\bar{n}$ in the intermediate state

In the low-density limit the relative annihilation probability of the intermediate antineutron r_1 is [18–20]

$$r_1 = \frac{\sigma_{\rm a}}{\sigma_{\rm t}} > 0.7 \,, \tag{28}$$

$\sigma_{\rm t} = \sigma_{\rm a} + \sigma_{\rm s}$, where $\sigma_{\rm a}$ and $\sigma_{\rm s}$ are the cross sections of free-space $\bar{n}N$ annihilation and $\bar{n}N$ scattering, respectively. The ratio (28) or (22) is very important for the correct construction of the model.

The model given above reproduces the magnitudes of r and r_1. Indeed, let us consider the free-space process

$$n + N \to \bar{n} + N \to f \,, \tag{29}$$

where f denotes $\bar{n}N$ or M. The annihilation and scattering channels are defined by (11) and

$$n + N \to \bar{n} + N \to \bar{n} + N \,, \tag{30}$$

respectively. The corresponding diagrams are shown in Figs. 1b and 6a. Using the amplitude (14), the cross section of process (11) is found to be

$$\sigma^{nN}_{\rm a} = N \int {\rm d}\Phi |M_{1{\rm b}}|^2 = a^2 N \int {\rm d}\Phi |M_{\rm a}|^2 = a^2 \sigma_{\rm a} \,, \tag{31}$$

$a = \epsilon G_0$. The normalization multiplier N is the same for $\sigma^{nN}_{\rm a}$ and $\sigma_{\rm a}$.

For process (30) a similar calculation gives

$$\sigma^{nN}_{\rm s} = a^2 \sigma_{\rm s} \tag{32}$$

and correspondingly

$$\frac{\sigma^{nN}_{\rm a}}{\sigma^{nN}_{\rm s}} = \frac{\sigma_{\rm a}}{\sigma_{\rm s}} = r \,. \tag{33}$$

The model (13) reproduces the ratio r.

For the $n\bar{n}$ transition in the medium the calculation will proceed by means of the optical theorem. To check this calculation we obtain r_1 for the free-space process (29) by

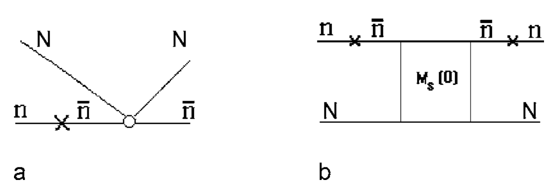

Fig. 6. Free-space processes $n+N \to \bar{n}+N \to \bar{n}+N$ (**a**) and $n+N \to \bar{n}+N \to n+N$ (**b**)

means of the optical theorem as well. The on-diagonal matrix element (see Fig. 6b) is

$$M(0) = \epsilon G_0 M_{\rm s}(0) G_0 \epsilon = a^2 M_{\rm s}(0)\,, \qquad (34)$$

where $M_{\rm s}(0)$ is the zero angle $\bar{n}N$ scattering amplitude. Let $\sigma_{\rm t}^{nN}$ be the total cross section of process (29). Using the optical theorem in the left- and right-hand sides of (34), we get

$$\sigma_{\rm t}^{nN} = a^2 \sigma_{\rm t} \qquad (35)$$

and

$$\frac{\sigma_{\rm a}^{nN}}{\sigma_{\rm t}^{nN}} = \frac{\sigma_{\rm a}}{\sigma_{\rm t}} = r_1\,. \qquad (36)$$

For process (29) the relative probability of the annihilation channel is given by (28), as we wished prove.

In the medium instead of (11) and (29) one should consider the process (2) and the inclusive $n\bar{n}$ transition

$$(n\text{–medium}) \to (\bar{n}\text{–medium}) \to f_m\,, \qquad (37)$$

respectively. Here f_m denotes M or $\bar{n}$. The result is the same (see Appendix A): for process (37) the relative annihilation probability of the intermediate $\bar{n}$ is given by (28).

The ratio (28) is explicitly used only in the classical models like the cascade one [21]. However, the Hamiltonian should contain all the information needed, which allows for the calculation of r or r_1. The fact that the model reproduces these ratios is very important; otherwise one can get a wrong, *additional* suppression as in model (20). Since the potential model does not describe the processes (11) and (2), it cannot reproduce (33) and (A.6).

The principal results of Sect. 4 are as follows. (a) The antineutron propagator is bare and singular. The particle self-energy and amplitudes of subprocesses should be *self-consistent.* (b) In the low-density limit the ratio (28) should be reproduced. This can be considered as a necessary condition for the correct construction of the model. Model (15) satisfies this requirement.

5 Field-theoretical approach with finite time interval

The model must satisfy the following requirements. a) The S-matrix should be unitary. b) The model should reproduce the free-space process shown in Fig. 1 and competition between scattering and annihilation considered above. These conditions are obvious; however, they are not fulfilled in the potential model.

The interaction Hamiltonian is given by (15). We use the basis $(n,\bar{n})$. The results do not depend on the basis. A main part of the existing calculations have been done in the n–$\bar{n}$ representation. The physics of the problem is in the Hamiltonian. The transition to the basis of stationary states is a formal step. It makes sense only in the case of the potential model $H = V = \text{const.}$, when the Hamiltonian of the $\bar{n}$–medium interaction is replaced by the effective mass $H \to m_{\rm eff} = {\rm Re}\,V - {\rm i}\Gamma/2$. Since the calculation of process (2) will be done beyond the potential model, the procedure of the diagonalization of the mass matrix is unrelated to our problem.

The S-matrix amplitudes corresponding to Figs. 1b and 2a are singular, as $G_0 \sim 1/0$ and $G_0^m \sim 1/0$. Contrary to quantum electrodynamics, the formal sum of the series in ϵ gives the meaningless self-energy $\Sigma \sim \epsilon^2/0$. This is because the Hamiltonian $\mathcal{H}_{n\bar{n}}$ corresponds to the 2-tail case. There is no compensation mechanism by radiative corrections.

For solving the problem, the FTA is used [17]. It is infrared-free. The calculation is performed by means of the evolution operator $U(t,0)$. The limiting transition $t \to \infty$ is not made, as it is physically incorrect. The FTA can be used for any problem, since for the non-singular diagrams it converts to the S-matrix approach (see Sect. 6.1).

5.1 $n\bar{n}$ transitions with $\bar{n}$ in the final state

First of all we consider the $n\bar{n}$ transitions with $\bar{n}$ in the final states on the *finite time interval* $(t,0)$ (see Fig. 7).

We introduce the evolution operator $U(t,0) = I + {\rm i}T(t,0)$. In the lowest order in ϵ, the matrix element $T_{\bar{n}n}$ is given by

$$\begin{aligned}\langle \bar{n}0|U(t,0) - I|0n\rangle &= {\rm i}T_{\bar{n}n}(t,0)\\ &= -{\rm i}\langle \bar{n}_p 0| \int_0^t {\rm d}t_c H_{n\bar{n}}(t_c)\\ &\quad + T^{\bar{n}}(t,0) \int_0^{t_k} {\rm d}t_c H_{n\bar{n}}(t_c)|0n_p\rangle\,,\end{aligned} \qquad (38)$$

$$\begin{aligned}T^{\bar{n}}(t,t_c) &= T\exp\left(-{\rm i}\int_{t_c}^t {\rm d}t_1 H(t_1)\right) - 1\\ &= \sum_{k=1}^{\infty}(-{\rm i})^k \int_{t_c}^t {\rm d}t_1 \cdots \int_{t_c}^{t_{k-1}} {\rm d}t_k H(t_1)\ldots H(t_k)\,,\end{aligned} \qquad (39)$$

where $|0n_p\rangle$ and $|0\bar{n}_p\rangle$ are the states of the medium containing the neutron and antineutron with the 4-momentum $p = (\epsilon_n, \mathbf{p}_n)$, respectively; $T^{\bar{n}}$ is the T-operator of the $\bar{n}$–medium interaction (compare with (17)).

We expand the Ψ-operators in the eigenfunctions of the unperturbed Hamiltonian $H_0 = -\nabla^2/2m + U_n$. Taking into account that $H_{n\bar{n}}|0n_p\rangle = \epsilon|0\bar{n}_p\rangle$, we change the order of integration [17] and obtain

$$\begin{aligned}T_{\bar{n}n}(t,0) &= -\epsilon t - \epsilon\int_0^t {\rm d}t_c {\rm i}T_{ii}^{\bar{n}}(t-t_c)\,,\\ {\rm i}T_{ii}^{\bar{n}}(\tau) &= \langle \bar{n}_p 0|T^{\bar{n}}(\tau)|0\bar{n}_p\rangle\,,\end{aligned} \qquad (40)$$

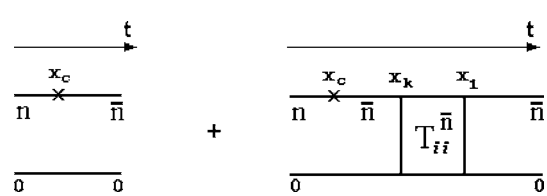

Fig. 7. $n\bar{n}$ transition in the medium with $\bar{n}$ in the final state

where $\tau = t - t_c$ and $T^{\bar{n}}(\tau) = T^{\bar{n}}(t, t_c)$. The $\bar{n}$–medium interaction is separated out in the block $T^{\bar{n}}_{ii}(\tau)$. This equation is important, since the structure of the matrix element corresponding to the process (2) is similar (see (64)). On the other hand, (40) can be verified with the use of the exactly solvable potential model.

5.2 Verification of FTA

To verify the FTA we obtain the results (5) and (6) of the potential model. As in Sect. 2, we take $H = V = \text{const}$. The block $T^{\bar{n}}_{ii}(\tau)$ is easily evaluated, resulting in

$$\mathrm{i}T^{\bar{n}}_{ii}(\tau) = U^{\bar{n}}_{ii}(\tau) - 1 = \exp(-\mathrm{i}V\tau) - 1\,. \qquad (41)$$

The probability of finding an $\bar{n}$ is

$$W_{\bar{n}}(t, 0) = |T_{\bar{n}n}(t, 0)|^2\,. \qquad (42)$$

By means of (40) and (41) it is easy to verify that $|T_{\bar{n}n}(t, 0)|^2$ coincides with (5).

The total $n\bar{n}$ transition probability W_t is given by

$$W_\mathrm{t}(t, 0) = 1 - |U_{ii}(t, 0)|^2 \approx 2\,\mathrm{Im}\,T_{ii}(t, 0)\,, \qquad (43)$$

where $U_{ii}(t, 0) = \langle n_p 0|U(t, 0)|0 n_p\rangle$. In the framework of the FTA the on-diagonal matrix element T_{ii} has been calculated in [17]:

$$T_{ii}(t, 0) = \mathrm{i}\epsilon^2 t^2/2 - \epsilon^2 \int_0^t \mathrm{d}t_\alpha \int_0^{t_\alpha} \mathrm{d}t_c T^{\bar{n}}_{ii}(t_\alpha - t_c)\,. \qquad (44)$$

Using (41) and (44), one obtains that $2\,\mathrm{Im}\,T_{ii} = W_\mathrm{t}^{\mathrm{pot}}$.

Consequently, the FTA reproduces all the results of the potential model. This was to be expected, since one and the same Hamiltonian was used. The same is also true for any ab transitions: $n\bar{n}$, $K^0\bar{K}^0$, neutrino oscillations. (The generalization for the relativistic case is trivial.)

5.3 Cancellation of divergences in the potential model

The consideration given above is infrared-free. Let us return to the S-matrix problem formulation $(\infty, -\infty)$. Due to the zero momentum transfer in the ϵ-vertex, any matrix element contains a singular propagator (see Figs. 7 and 8a). However, the matrix element of the potential model T_{ii} obtained by means of the S-matrix approach is not singular (see (9)). The same is true for the process shown in Fig. 7. From the standpoint of microscopic theory the reason is as follows.

In addition to the singular propagator the matrix elements mentioned above also contain the block $T^{\bar{n}}_{ii}$, which is a sum of the zero angle rescattering diagrams of $\bar{n}$. As a result, the self-energy part $\Sigma = V$ appears. The corresponding mechanism of the cancellation of divergences (the forming of the self-energy part) is illustrated by (19), where $G^m_0 \sim 1/0$.

We are interested in the off-diagonal matrix elements that do not contain the sum mentioned above ($T^{\bar{n}}_{f\neq i}$ instead of $T^{\bar{n}}_{ii}$), which, hence, diverges because one singular propagator after the ϵ-vertex appears in any case. (Recall that the formal sum of the series in ϵ gives the meaningless self-energy part $\Sigma \sim \epsilon^2/0$.)

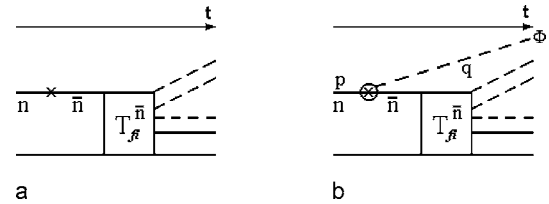

Fig. 8. **a** $n\bar{n}$ transition in the medium followed by annihilation. **b** Same as **a** but with escaping of particle in the $n\bar{n}$ transition vertex

The principal result of this section is that the FTA has been verified by the example of the exactly solvable potential model. It is involved in the block $\mathrm{i}T^{\bar{n}}_{ii}(\tau) = \langle \bar{n}_p 0|T^{\bar{n}}(\tau)|0\bar{n}_p\rangle$ as a special case.

6 $n\bar{n}$ transitions followed by annihilation

As shown above, the FTA reproduces all the potential model results. Besides, for non-singular diagrams it converts to the S-matrix theory (see Sect. 6.1). We now proceed to the main calculation.

Let us consider the process (2) in nuclear matter (see Fig. 8a). The Hamiltonians H_0 and $H_\mathrm{I}(t)$ are the same as in Sect. 4. The 4-momenta of n and $\bar{n}$ coincide. The $T^{\bar{n}}$-operator involves all the $\bar{n}$–medium interactions. In consequence of this, $\Sigma = 0$. In essence, we deal with two-step nuclear decay: dynamical $n\bar{n}$ conversion and annihilation. Its dynamical part lasts only 10^{-24} s. The sole distinction with respect to the decay theory is that the FTA should be used, because the antineutron propagator is singular.

We give the expressions for the amplitudes from [16]. Next they will be obtained as a special case of a more general problem. The matrix element of the process shown in Fig. 8a is

$$T_{fi}(t) = -\epsilon \int_0^t \mathrm{d}t_c \mathrm{i}T^{\bar{n}}_{fi}(t, t_c)\,, \qquad (45)$$

$$\mathrm{i}T^{\bar{n}}_{fi}(t, t_c) = \mathrm{i}T^{\bar{n}}_{fi}(\tau) = \langle f|T^{\bar{n}}(\tau)|0\bar{n}_p\rangle. \qquad (46)$$

Here $T^{\bar{n}}_{fi}(\tau)$ is the matrix element of the antineutron annihilation in a time $\tau = t - t_c$ (compare with the matrix element of the S-matrix in (17)). The $T^{\bar{n}}(\tau)$-operator is given by (39). Similarly to (40), the $\bar{n}$–medium annihilation is separated out in the block $T^{\bar{n}}_{fi}(\tau)$.

Consider now the more general problem. We calculate the matrix element $T_{fi}(t)$ shown in Fig. 8b on the interval $(t/2, -t/2)$. As a result, the following will be shown. (a) If $q \neq 0$ (q is the 4-momentum of particle escaped in the $n\bar{n}$ transition vertex) and $t \to \infty$, we come to the usual S-matrix amplitude. (b) If $q \to 0$, (45) is obtained. Such a scheme allows one to verify and study the FTA. Also we will see the point in which the standard calculation scheme should be changed. (Some preliminary calculations are sketched in [22].)

Consider the imaginary free-space decay

$$n \to \bar{n} + \Phi\,, \tag{47}$$

$\Phi(x) = N_\Phi \exp(-\mathrm{i}qx)$, where $N_\Phi = (2q_0\Omega)^{-1/2}$. For decay to be permissible in vacuum put $m_{\bar{n}} = m - 2m_\Phi$. As with $\mathcal{H}_{n\bar{n}}$, the decay Hamiltonian $\mathcal{H}'_{n\bar{n}}$ is taken in the scalar form $\mathcal{H}'_{n\bar{n}} = \epsilon' \bar{\Psi}_{\bar{n}} \Phi^* \Psi_n + \mathrm{H.c.}$

The corresponding process in nuclear matter is shown in Fig. 8b. This is the nearest analogy to the process under study. The neutron wave function is $n_p(x) = N_n \exp(-\mathrm{i}px)$, where $N_n = \Omega^{-1/2}$, $p = (p_0, \mathbf{p})$ and $p_0 = m + \mathbf{p}^2/2m$. The background nuclear matter field U_n is omitted.

Instead of (15) we have

$$H_\mathrm{I} = H'_{n\bar{n}} + H\,, \qquad H'_{n\bar{n}}(t) = \epsilon' \int \mathrm{d}^3x \big(\bar{\Psi}_{\bar{n}} \Phi^* \Psi_n + \mathrm{H.c.}\big); \tag{48}$$

ϵ' is dimensionless. In the lowest order in $H'_{n\bar{n}}$, the matrix element $T_{fi}(t)$ is

$$T_{fi}(t) = -\langle \Phi_q f0| \sum_{k=1}^{\infty} T_k(t) \int_{-t/2}^{t_k} \mathrm{d}t_c H'_{n\bar{n}}(t_c)|0n_p\rangle\,,$$
$$T_k(t) = (-\mathrm{i})^k \int_{-t/2}^{t/2} \mathrm{d}t_1 \cdots \int_{-t/2}^{t_{k-1}} \mathrm{d}t_k H(t_1)\ldots H(t_k)\,. \tag{49}$$

Here $\langle f|$ represents the annihilation products with (n) mesons. For the 3-tail $\mathcal{H}'_{n\bar{n}}$, the relation $H_{n\bar{n}}|0n_p\rangle = \epsilon|0\bar{n}_p\rangle$ used in Sect. 5.1 is invalid. A direct calculation is needed.

Using the standard rules of quantum field theory, we obtain (see Appendix B)

$$T_{fi}(t) = -\mathrm{i}\epsilon' N_n N_\Phi \langle f0| \sum_{k=1}^{\infty} (-\mathrm{i}) T_{k-1}(t) \int_{-t/2}^{t_{k-1}} \mathrm{d}t_k \times \int \mathrm{d}^3x_k \mathcal{H}'(x_k) \mathrm{e}^{\mathrm{i}(\mathbf{p}-\mathbf{q})\mathbf{x}_k} I(t_k)|0\rangle\,, \tag{50}$$

$$I(t_k) = \int_{-\infty}^{\infty} \frac{\mathrm{d}k_0}{2\pi} \int_{-t/2}^{t_k} \mathrm{d}t_c \frac{1}{k_0 - m_{\bar{n}} - (\mathbf{p}-\mathbf{q})^2/2m_{\bar{n}} + \mathrm{i}0} \times \mathrm{e}^{-\mathrm{i}k_0 t_k} \mathrm{e}^{\mathrm{i}t_c(q_0 - p_0 + k_0)}\,. \tag{51}$$

From this point on the calculations for Fig. 8a and b are essentially different. In (51) we put $t_k = \infty$ and $-t/2 = -\infty$. Then

$$I(\infty) = \int \frac{\mathrm{d}k_0}{2\pi} \frac{\mathrm{e}^{-\mathrm{i}k_0 t_k}}{k_0 - m_{\bar{n}} - (\mathbf{p}-\mathbf{q})^2/2m_{\bar{n}} + \mathrm{i}0} \times \int_{-\infty}^{\infty} \mathrm{d}t_c \mathrm{e}^{\mathrm{i}t_c(q_0 - p_0 + k_0)} \tag{52}$$

and correspondingly

$$I(\infty) = G\mathrm{e}^{-\mathrm{i}(p_0 - q_0)t_k}\,, \tag{53}$$

$$G = \frac{1}{p_0 - q_0 - m_{\bar{n}} - (\mathbf{p}-\mathbf{q})^2/2m_{\bar{n}} + \mathrm{i}0}\,, \tag{54}$$

where G is the non-relativistic antineutron propagator.

Let $q = (0, 0)$ and $m_{\bar{n}} = m$ (see Fig. 8a). Now

$$G = \frac{1}{p_0 - m - \mathbf{p}^2/2m} \sim \frac{1}{0} \tag{55}$$

and $T_{fi} \sim 1/0$. This is an irremovable peculiarity. We deal with an infrared divergence, which is obvious from Fig. 8a. We thus see the specific point (the limiting transition $t \to \infty$ in (50)) in which the standard S-matrix scheme should be changed.

6.1 Non-singular diagram

Let us obtain now the amplitudes corresponding to Fig. 8a and b starting from (50). If $q \neq 0$, the limit $t \to \infty$ can be considered. In (50) we put $t \to \infty$ and substitute (53). Taking into account that

$$N_{\bar{n}} \mathrm{e}^{-\mathrm{i}(p-q)x_k}|0\rangle = \Psi_{\bar{n}}(x_k)|\bar{n}_{p-q}\rangle \tag{56}$$

and using the relation $\int \mathrm{d}^3x_k \mathcal{H}'(x_k)\Psi_{\bar{n}}(x_k) = H(t_k)$ (see Appendix B), one obtains

$$T_{fi} = -\mathrm{i}N_\Phi \epsilon' G \langle f0|T^{\bar{n}}|0\bar{n}_{p-q}\rangle\,. \tag{57}$$

Here $|0\bar{n}_{p-q}\rangle$ is the state of the medium containing the $\bar{n}$ with the 4-momentum $p-q$, and the $T^{\bar{n}}$-operator is given by (17). With the help of the relation

$$\mathrm{i}T_{fi} = \langle f|T|i\rangle = N(2\pi)^4\delta^4(p_f - p_i)M_{fi}$$

we rewrite (57) in terms of the amplitudes

$$M_{8\mathrm{b}} = \epsilon' G M_\mathrm{a}^m\,. \tag{58}$$

Here $M_{8\mathrm{b}}$ is the amplitude of the process shown in Fig. 8b, M_a^m is the annihilation amplitude of $\bar{n}$ with the 4-momentum $p-q$ and G is given by (54). We have obtained the usual S-matrix amplitude, which is the verification of (50). As in (16), the antineutron propagator is bare.

It is easy to estimate the widths corresponding to Fig. 8b and the free-space decay (47):

$$\Gamma_{8\mathrm{b}} \approx \epsilon'^2 \Gamma/(2\pi^2)\,, \qquad \Gamma_\mathrm{free} \approx \epsilon'^2 m_\Phi/(2\pi)\,, \tag{59}$$

where we have put $m_\Phi/m \ll 1$. The t-dependence is determined by the exponential decay law:

$$W_{8\mathrm{b,free}} = 1 - \mathrm{e}^{-\Gamma_{8\mathrm{b,free}}t} \sim \Gamma_{8\mathrm{b,free}}t\,. \tag{60}$$

These formulas will be needed below.

6.2 Singular diagram

Let $q=0$ and $m_{\bar{n}}=m$ (see Fig. 8a). In (50) one should put $\epsilon'=\epsilon$ and $N_\Phi=1$. Upon integration with respect to k_0, (51) becomes

$$I(t_k)=-\mathrm{i}\mathrm{e}^{-\mathrm{i}t_k p_0}\int_{-t/2}^{t_k}\mathrm{d}t_c\,. \quad (61)$$

As in (56), $N_{\bar{n}}\exp(-\mathrm{i}px_k)|0\rangle=\Psi_{\bar{n}}(x_k)|\bar{n}_p\rangle$. Turning back to the Hamiltonian $H(t_k)$, one obtains

$$T_{fi}(t)=-\mathrm{i}\epsilon\langle f0|\sum_{k=1}^{\infty}T_k(t)\int_{-t/2}^{t_k}\mathrm{d}t_c|0\bar{n}_p\rangle\,. \quad (62)$$

Using the formula

$$\int_{-t/2}^{t/2}\mathrm{d}t_1\cdots\int_{-t/2}^{t_{k-1}}\mathrm{d}t_k\int_{-t/2}^{t_k}\mathrm{d}t_c f(t_1,\ldots,t_c)$$
$$=\int_{-t/2}^{t/2}\mathrm{d}t_c\int_{t_c}^{t/2}\mathrm{d}t_1\cdots\int_{t_c}^{t_{k-1}}\mathrm{d}t_k f(t_1,\ldots,t_c)\,, \quad (63)$$

we change the integration order and pass on to the interval $(t,0)$. Finally

$$T_{fi}(t)=-\epsilon\int_0^t\mathrm{d}t_c\langle f0|T^{\bar{n}}(t-t_c)|0\bar{n}_p\rangle\,,$$
$$\langle f0|T^{\bar{n}}(\tau)|0\bar{n}_p\rangle=\mathrm{i}T^{\bar{n}}_{fi}(\tau)\,, \quad (64)$$

which coincides with (45). The result is expressed through the submatrix $T^{\bar{n}}_{fi}(\tau)$. (Compare with (40).) Note that $T_{fi}(t)$ coincides with the second term of (40) with the replacement $\langle i|=\langle\bar{n}_p0|\rightarrow\langle f|$. This can be considered as a test for the $T_{fi}(t)$.

Comparing (64) and (57), one can see the formal correspondence: if $q\rightarrow 0$, $GT^{\bar{n}}_{fi}\rightarrow \mathrm{i}\int_0^t\mathrm{d}\tau T^{\bar{n}}_{fi}(\tau)$.

7 Infrared singularities and the formulation of the S-matrix problem

In this section we consider the time-dependence of the matrix elements and the other characteristic features of the FTA and complete the calculation of process (2).

The FTA is infrared-free. This approach is naturally connected with the experimental conditions. Indeed, measurement of any process corresponds to some interval τ. So it is necessary to calculate $U_{fi}(\tau)$. The replacement $U(\tau)\rightarrow S(\infty)$ is justified if the main contribution gives some region $\Delta<\tau$, so that $U_{fi}(\tau>\Delta)=U_{fi}(\infty)=S_{fi}=$ const. The expressions of this type are the basis for all S-matrix calculations. The following cases are possible.

1. There is bound to be an asymptotic regime. Then the usual scheme realized in field theory or non-stationary theory of scattering is relevant. Figure 8b corresponds to this case.

2. There is no asymptotic regime. An example is provided by the oscillation Hamiltonian $\mathcal{H}_{n\bar{n}}$. We have the usual non-stationary problem. The S-matrix approach is inapplicable. Because of this, for Fig. 8a the calculation has been done in the framework of FTA.

A somewhat different explanation of the application of the FTA is as follows. If $\mathcal{H}_\mathrm{I}=\mathcal{H}_{n\bar{n}}$, the solution is periodic. It is obtained by means of non-stationary equations of motion and not S-matrix theory. This is clear from the definition of the S-matrix. To reproduce the limiting case $\mathcal{H}\rightarrow 0$, i.e. the periodic solution, we have to use the FTA.

Let us return to (64). The annihilation of $\bar{n}$ in nuclear matter can be considered as the decay of a one-particle state with the characteristic time τ_a. Correspondingly, $T^{\bar{n}}_{fi}$ can be interpreted as the decay matrix of the $\bar{n}$–medium state. Obviously

$$T^{\bar{n}}_{fi}(\tau>\tau_\mathrm{a})\approx T^{\bar{n}}_{fi}=\mathrm{const.} \quad (65)$$

and

$$W^{\bar{n}}=\sum_{f\neq i}\left|T^{\bar{n}}_{fi}\right|^2=1\,, \quad (66)$$

where $W^{\bar{n}}$ is the total decay probability of the $\bar{n}$–nucleus. Let

$$t\gg\Delta\approx\tau_\mathrm{a}\,. \quad (67)$$

In view of this condition the submatrix $T^{\bar{n}}_{fi}$ can be calculated by means of S-matrix theory. The FTA is needed only for the description of the subprocess of the $n\bar{n}$ conversion. However, the condition (66) greatly simplifies the calculation. One can write immediately [16]

$$W_\mathrm{a}(t)\approx\sum_{f\neq i}\left|-\mathrm{i}\epsilon tT^{\bar{n}}_{fi}\right|^2=\epsilon^2t^2W^{\bar{n}}=\epsilon^2t^2=W_f(t)\,, \quad (68)$$

where $W_\mathrm{a}(t)$ is the probability of the process (2).

For $n\bar{n}$ transitions in nuclear W_t, $W_\mathrm{t}(t)=W_\mathrm{a}(t)$, since all the $\bar{n}$ annihilate. The interpretation of $W_\mathrm{a}(t)$ has been given above: momentary $n\bar{n}$ conversion at some point in time between 0 and t; annihilation in a time $\tau_\mathrm{a}\sim 6\times 10^{-24}$ s. The explanation of the t^2-dependence is simple. The process shown in Fig. 8a represents two consecutive subprocesses. The speed and probability of the whole process are defined by those of the slower subprocess. Since $\tau_\mathrm{a}\ll t$, the annihilation can be considered instantaneous: for any $t_1<t$ the annihilation probability is $W^{\bar{n}}(t-t_1)\approx 1$. So, the probability of process (2) is defined by the speed of the $n\bar{n}$ transition: $W_\mathrm{a}\approx W_f\sim t^2$, but not $\sim t/\Gamma$ (see (6)). In essence, we deal with the limiting case $\tau/t\rightarrow 0$, or, similarly, $T^{\bar{n}}_{fi}(\tau)=T^{\bar{n}}_{fi}$ at any τ. Formally, the quadratic time-dependence follows from (64).

Owing to the annihilation channel, W_a is practically equal to the free-space $n\bar{n}$ transition probability. The fact that $\Sigma=0$ tends to increase W_a. So, $\tau_{n\bar{n}}\sim T_{n\bar{n}}$, where $T_{n\bar{n}}$ is the oscillation time of the neutron bound in a nucleus.

All the results have been obtained by means of formal expansions. They are valid at any finite t. Consequently, the singularities of the S-matrix amplitudes $M_{1\mathrm{b}}$ and M_2

result from the erroneous formulation of the problem. The problem should be formulated on the finite interval $(t, 0)$. If $t \to \infty$, (68) diverges just as the modulus (16) squared does. The infrared singularities point to the fact that there is no asymptotic regime.

8 Summary and discussion

The importance of the unitarity condition is well known [23–25]. Nevertheless, non-Hermitian models are frequently used, because on the one hand, they greatly simplify the calculation and on the other hand, it is hoped that an error may be inessential. This paper demonstrates that the non-unitarity of the S-matrix can produce a qualitative error in the results. Certainly, unitarity is a necessary and not a sufficient condition. We compare our results and the one of the potential model.

The time-dependence is a more important characteristic of any process. It is common knowledge that the t-dependence of the decay probability in the vacuum and medium is identical. Equation (60) illustrates this fact. In our calculation the t-dependencies coincide as well: $W_{\rm a} \sim W_{\rm t} \sim t^2$ and $W_f \sim t^2$. The potential model gives $W_{\rm t}^{\rm pot} \sim t$, whereas $W_f \sim t^2$. There is no reason known why we have such a fundamental change.

The Γ-dependence of the results differs fundamentally as well. The probability of the decay shown in Fig. 8b is linear in Γ:

$$W_{\rm 8b} = \Gamma_{\rm 8b} t \sim \Gamma t \qquad (69)$$

(see (59) and (60)). For Fig. 8a the annihilation effect acts in the same direction

$$W_{\rm a} \sim W^{\bar{n}} \sim \Gamma \,. \qquad (70)$$

In the potential model the effect of absorption acts in the opposite direction, $W_{\rm t}^{\rm pot} \sim 1/\Gamma$. Recall that the annihilation is the basic effect determining the speed of the process (see (6) and (68)).

Let us consider the suppression factor R. From (68) we have

$$R = \frac{W_{\rm a}}{W_f} \sim 1 \,. \qquad (71)$$

For similar processes the value $R \sim 1$ is typical. Indeed, in the medium the free-space decay (47) is suppressed by the factor

$$\frac{\Gamma_{\rm 8b}}{\Gamma_{\rm free}} = \frac{\Gamma}{\pi m_{\Phi}} \approx \frac{1}{\pi} \,, \qquad (72)$$

where we have put $m_{\Phi} \approx \Gamma$.

A realistic example is the pion production $pn \to pp\pi^-$ in vacuum and on a neutron bound in a nucleus. If the pion energy is in the region of resonance, the pion absorption is very strong. This has effects on the number of pions emitted from the nucleus, but not on the fact of pion formation inside the nucleus. (In the latter case the pion and the products of pion absorption should be detected).

In the processes cited above $R \sim 1$. The potential model gives $R_{\rm pot} \to 0$: if $\Gamma \sim 100\,{\rm MeV}$ and $t \sim 1\,{\rm yr}$ [26], $R_{\rm pot} \sim 10^{-30}$.

Consequently, in the potential model the t- and Γ-dependencies are principally incorrect. As a result, the suppression is enormous: $R_{\rm pot} \to 0$. This is not surprising, since the potential model describes only $W_{\bar{n}}$. Recall that in the strong absorption region $W_{\bar{n}} \ll W_{\rm a}$.

The next important point is the competition between scattering and annihilation in the intermediate state. The models (13) and (15) reproduce the values of r and r_1 (see Sect. 4.3). Since the potential model does not describe the processes (2) and (11), it makes no sense to speak of a competition effect in this model. The greater the $|\,{\rm Im}\, V|$, the greater the error in the $W_{\rm t}^{\rm pot}$ and $W_{\rm a}$ calculated by means of the potential model.

Consider now the effects of coherent forward scattering and absorption. Let there be forward scattering alone: $H = {\rm Re}\, V$. Since the FTA reproduces all the potential model results (see Sect. 5.2), it describes the above-mentioned special case as well, and in particular the suppression of oscillations by ${\rm Re}\, V$.

Let there be an annihilation vertex only: $V = 0$ and

$$\mathcal{H} = \mathcal{H}_{\rm a} \,. \qquad (73)$$

The annihilation Hamiltonian $\mathcal{H}_{\rm a}$ is given by (27). In this case we inevitably arrive at the amplitude with singular propagator. The dressed propagator cannot arise in principal (see Sect. 4.2). In view of (22) the model (73) is *reasonable* and so the result $W_{\rm a} \approx W_f$ seems quite natural for us. In our calculation the approximation (73) has not been used. Nevertheless, the result is the same as in model (73). In this connection we briefly outline the principal points of our calculation.

The process shown in Fig. 8b is described by the Hamiltonian $H_{\rm I} = H'_{n\bar{n}} + H$. Since H appears in the block $T^{\bar{n}}_{fi}$ only, the antineutron propagator is bare. For Fig. 8a the picture is the same; however, $T_{fi} \sim 1/0$ (here we keep in mind the S-matrix problem formulation). Due to this we had to use the FTA. It must be emphasized that in the potential model the antineutron propagator between ϵ-vertex and block $T^{\bar{n}}_{ii}$ is also bare (see Fig. 7 and Sect. 5.3), since the interaction Hamiltonian has the same structure (15).

The fact that antineutron propagator is bare is principal. It entails the divergence of the S-matrix amplitude; the application of FTA; the linear time-dependence of the matrix element $T_{fi}(t)$ and the t^2-dependence of the result. As a result $W_{\rm a}$ increases. In our opinion the models with a dressed (and consequently non-singular) propagator are non-realistic (see Sect. 4).

(Recall that in the potential model $\Sigma = V$ by the construction of the model. Since this model is inapplicable, the field-theoretical approach is used. The self-energy should be considered in the context of the concrete problem. Obviously, for Fig. 8b the propagator is bare. For Fig. 8a it is bare as well, because the $\bar{n}$–medium interaction is the *same*.)

All the formulas up to (64) are true for any ab transitions in which $m_\mathrm{a} = m_b$. (Generalizations for the relativistic case and the case when $m_\mathrm{a} \neq m_b$ are simple.) The next important point is the condition (67). For $n\bar{n}$ transitions in nuclei it is obvious, because in this case the value $t = T_0 = 1.3\,\mathrm{yr}$ [26] is used (T_0 is the observation time in a proton decay-type experiment). The condition $t \gg \tau_\mathrm{a}$ leads to (65) and (66). Due to these, the result does not depend on the specific form of H and coincides with the result given by the model (73).

Once the antineutron annihilation amplitude is defined by (46), the rest of the calculation is rather formal. The distinguishing feature of the model is that the amplitude of the process is "proportional" to the annihilation amplitude $T_{fi} \sim T^{\bar{n}}_{fi}$. This structure is typical for the direct processes.

If the condition $t \gg \Delta$ is not fulfilled, a direct calculation of (64) is needed. However, the qualitative picture remains the same: the process amplitude is proportional to the absorption amplitude.

It is interesting to study the behavior of W_t in the intermediate range, $t \sim \Delta$. It seems plausible that W_t depends slightly on the value of Δ/t (in comparison with the potential model results). We also note that there is no asymptotic regime for free-space $K^0\bar{K}^0$ oscillations. In our opinion, it makes sense to look at the calculation of $\Delta m = m_\mathrm{L} - m_\mathrm{S}$ (GIM mechanism) from the standpoint of the applicability of the S-matrix approach in this case (see Sect. 7).

9 Conclusion

The approach considered above reproduces all the results on the particle oscillations (Sect. 5.2). Certainly, for problems where the absorption is inessential, the standard model of oscillations is more handy, since it is more simple. Our approach is oriented to the processes like (1), which are not described by the potential model.

A direct calculation of $n\bar{n}$ transitions in nuclear matter followed by annihilation has been done. The results have been discussed in Sect. 8. We confirm our restriction [16] on the free-space $n\bar{n}$ oscillation time $\tau_{n\bar{n}} > 10^{16}$ yr. Compared to [16], the result (68) was obtained as a special case of a more general problem. Besides, the medium corrections, the uncertainties related to the amplitudes, the limiting cases and competition between scattering and annihilation in the intermediate state have been studied. Model (73) and the analysis made in Sects. 4 and 5.3 show that $\Sigma = 0$. Nevertheless, this is a point of great nicety. Further investigations are desirable. The region $t < \Delta$ and oscillations of other particles can be considered as well.

The calculation up to (64) is formal. With the replacement $T^{\bar{n}}_{fi}(\tau) \to T^b_{fi}(\tau)$, where T^b_{fi} is the b-particle absorption amplitude, the matrix element (64) describes the process (1) in which $m_a = m_b$. In this connection we point out some features of (64). a) The amplitude $T_{fi}(t)$ is "proportional" to the amplitude $T^b_{fi}(\tau)$. In the potential model the effect of b-particle absorption acts in the opposite direction, which tends to suppress the process. b) In the lowest order in ϵ the potential model gives the linear t-dependence $W^\mathrm{pot}_\mathrm{t} \sim t/\Gamma$. For any block $T^b_{fi}(\tau)$ model the time-dependence of the value $|T_{fi}(t)|^2$ cannot be linear.

Certain of the above-given results have a general nature. They are valid for any reaction or decay in the medium. The main ones of them are as follows. 1) For processes with zero momentum transfer the problem should be formulated on a finite time interval. 2) The final-state absorption (decay) does not tend to suppress the total process probability as well as the probability of the channel corresponding to absorption. This is true for the reactions, decays and ab transitions in the medium. 3) For an intermediate-state particle the low-density limit can be used. This enables one to verify the model. In particular, the particle self-energy and amplitudes of subprocesses should be self-consistent. In the lower-density approximation this problem becomes transparent.

Appendix A

In this appendix the relative annihilation probability of the intermediate $\bar{n}$ for $n\bar{n}$ transition in the medium is calculated. Similarly to (31), we obtain the probability of process (2) in a unit of time:

$$\Gamma_2 = N_1 \int \mathrm{d}\Phi |M_2|^2 = a_m^2 N_1 \int \mathrm{d}\Phi |M^m_\mathrm{a}|^2 = a_m^2 \Gamma\,, \tag{A.1}$$

$a_m = \epsilon G^m_0$. The normalization multiplier N_1 is the same for Γ_2 and Γ. The term "width" is not used, because the t-dependence of process (2) does not need to be $\exp(-\Gamma_2 t)$ (see Sect. 7).

In the low-density approximation [27, 28] $\Gamma = v\rho\sigma_\mathrm{a}$ and

$$\Gamma_2 = a_m^2 v\rho\sigma_\mathrm{a}\,. \tag{A.2}$$

The on-diagonal matrix element $M^m(0)$ corresponding to the process (n–medium) $\to$ ($\bar{n}$–medium) $\to$ (n–medium) is

$$M^m(0) = \epsilon G^m_0 M^m_\mathrm{s}(0) G^m_0 \epsilon = a_m^2 M^m_\mathrm{s}(0) \tag{A.3}$$

(compare with (34)). Here $M^m_\mathrm{s}(0)$ is the amplitude of zero angle scattering of $\bar{n}$ in the *medium*.

Taking into account that

$$\frac{1}{T_0} 2\,\mathrm{Im}\, M^m(0) = \Gamma_\mathrm{t}\,,$$
$$\frac{1}{T_0} 2\,\mathrm{Im}\, M^m_\mathrm{s}(0) = v\rho\sigma_\mathrm{t} \tag{A.4}$$

(T_0 is the normalization time and Γ_t is the probability of the process (37) in a unit of time), one obtains

$$\Gamma_\mathrm{t} = a_m^2 v\rho\sigma_\mathrm{t} \tag{A.5}$$

and correspondingly

$$\frac{\Gamma_2}{\Gamma_\mathrm{t}} = r_1 \,. \tag{A.6}$$

Equations (A.2) and (A.5) are interpreted in line with the low-density approximation physics.

Appendix B

The calculation is standard [29, 30] up to the integration over t. The neutron and antineutron are assumed to be spinless. We have

$$\Psi_n(x)|n_p\rangle = \Psi_n(x)b^+(p)|0\rangle = N_n \mathrm{e}^{-\mathrm{i}px}|0\rangle \,, \tag{B.1}$$

$$\langle\Phi_q|\Phi^*(x) = \langle 0|N_\Phi \mathrm{e}^{\mathrm{i}qx} \,. \tag{B.2}$$

Then

$$\langle\Phi_q|\Phi^*(x_c)\Psi_{\bar{n}}(x_c)|n_p\rangle = N_n N_\Phi \mathrm{e}^{\mathrm{i}(q-p)x_c} \,. \tag{B.3}$$

In the last multiplier of (49) we separate out the antineutron field operator $\Psi_{\bar{n}}(x_k)$:

$$H(t_k) = \int \mathrm{d}^3x_k \mathcal{H}(x_k) = \int \mathrm{d}^3x_k \mathcal{H}'(x_k)\Psi_{\bar{n}}(x_k) \,. \tag{B.4}$$

Equation (49) becomes

$$T_{fi}(t) = -\mathrm{i}\epsilon' N_n N_\Phi \langle f0| \sum_{k=1}^{\infty}(-\mathrm{i})T_{k-1}(t) \int_{-t/2}^{t_{k-1}} \mathrm{d}t_k \times \int \mathrm{d}^3x_k \mathcal{H}'(x_k)J(t_k)|0\rangle \,, \tag{B.5}$$

$$J(t_k) = \int_{-t/2}^{t_k} \mathrm{d}t_c \int \mathrm{d}^3x_c \langle T(\Psi_{\bar{n}}(x_k)\bar{\Psi}_{\bar{n}}(x_c))\rangle \mathrm{e}^{\mathrm{i}(q-p)x_c} \,. \tag{B.6}$$

For Fig. 8a the problem is non-relativistic and so for Fig. 8b we also take the non-relativistic antineutron propagator

$$\langle T(\Psi_{\bar{n}}(x_k)\bar{\Psi}_{\bar{n}}(x_c))\rangle = \mathrm{i}G(x_k - x_c) = \mathrm{i}\int \frac{\mathrm{d}k_0}{2\pi}\mathrm{e}^{-\mathrm{i}k_0(t_k-t_c)} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}(\mathbf{x}_k-\mathbf{x}_c)}}{k_0 - m_{\bar{n}} - \mathbf{k}^2/2m_{\bar{n}} + \mathrm{i}0} \,. \tag{B.7}$$

Upon integrating over $\mathbf{x}_c$ and $\mathbf{k}$ we obtain (50) and (51).

References

1. M.L. Good, Phys. Rev. **106**, 591 (1957)
2. M.L. Good, Phys. Rev. **110**, 550 (1958)
3. L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978)
4. E.D. Commins, P.H. Bucksbaum, Weak Interactions of Leptons and Quarks (Cambridge University Press, Cambridge, 1983)
5. F. Boehm, P. Vogel, Physics of Massive Neutrinos (Cambridge University Press, Cambridge, 1987)
6. V.I. Nazaruk, Mod. Phys. Lett. A **21**, 2189 (2006)
7. V.I. Nazaruk, Proc. of the 9th International Conference on Low Energy Antiproton Physics (LEAP'05), Bonn, Germany, 2005, ed. by D. Grzonka, R. Czyzykiewich, W. Oelert, T. Rozek, P. Winter, AIP Conf. Proc. **796**, 180 (2005)
8. V.I. Nazaruk, Eur. Phys. J. A **31**, 177 (2007)
9. V.A. Kuzmin, JETP Lett. **12**, 228 (1970)
10. S.L. Glashow, preprint HUTP-79/A059 (Harvard, 1979)
11. R.N. Mohapatra, R.E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980)
12. P.G.H. Sandars, J. Phys. G **6**, L161 (1980)
13. M.V. Kazarnovsky et al., Pisma. Zh. Eksp. Teor. Fiz. **32**, 88 (1980)
14. K.G. Chetyrkin et al., Phys. Lett. B **99**, 358 (1981)
15. J. Arafune, O. Miyamura, Prog. Theor. Phys. **66**, 661 (1981)
16. V.I. Nazaruk, Phys. Rev. C **58**, R1884 (1998)
17. V.I. Nazaruk, Phys. Lett. B **337**, 328 (1994)
18. T. Kalogeropouls, G.S. Tzanakos, Phys. Rev. D **22**, 2585 (1980)
19. G.S. Mutchlev et al., Phys. Rev. D **38**, 742 (1988)
20. W. Bruckner et al., preprint CERN-EP/89-105 (Geneva, 1989)
21. A.S. Iljinov, V.I. Nazaruk, S.E. Chigrinov, Nucl. Phys. A **382**, 378 (1982)
22. V.I. Nazaruk, Proc. of the 7th Conference on the Intersections of Particle and Nuclear Physics (CIPANP 2000), Quebec City, Canada, 2000, ed. by Z. Parsa, W.J. Marciano, AIP Conf. Proc. **549**, 901 (2000)
23. T. Inone, E. Oset, Nucl. Phys. A **710**, 354 (2002)
24. T. Inone, E. Oset, Nucl. Phys. A **721**, 661 (2003)
25. E. Oset, Nucl. Phys. A **721**, 58 (2003)
26. H. Takita et al., Phys. Rev. D **34**, 902 (1986)
27. C.B. Dover, J. Hufner, R.H. Lemmer, Ann. Phys. **66**, 248 (1971)
28. J. Hufner, Phys. Rep. **21**, 1 (1975)
29. J.D. Bjorken, S.D. Drell, Relativistic Quantum Fields (McGraw-Hill, New York, 1964)
30. T. Ericson, W. Weise, Pions and Nuclei (Clarendon Press, Oxford, 1988)